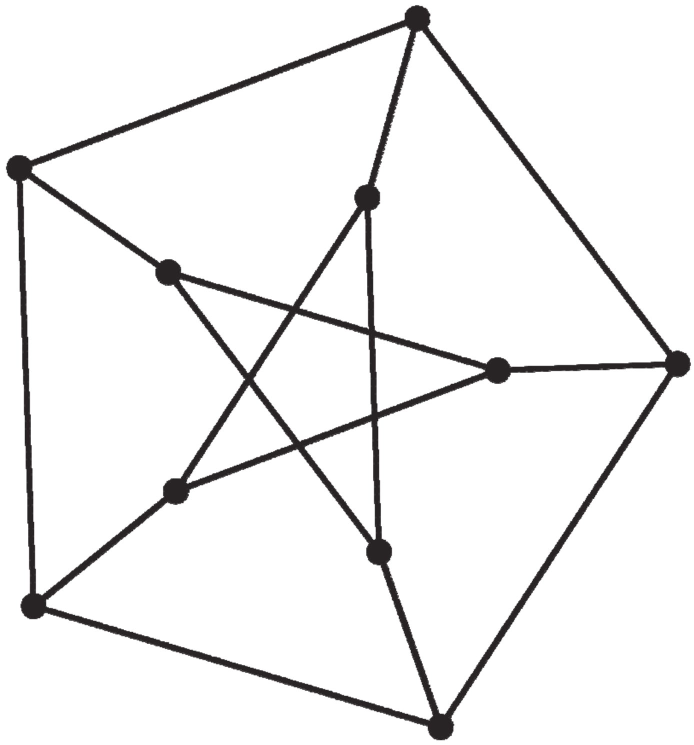
2008 年組合數學 暨 新苗研討會

報告摘要

2008 年 8 月 11 日至 8 月 12 日



08/11-12 '08
組合數學第17屆 新苗研討會
慶祝李國偉教授60歲生日·交通大學應用數學系

主辦單位：國立交通大學應用數學系

贊助單位：國科會數學研究推動中心

2008 年組合數學 暨 新苗研討會

謹獻於 李國偉老師六十歲生日

2008 年 8 月 11 日至 8 月 12 日

新竹市國立交通大學科學二館演講廳

http://jupiter.math.nctu.edu.tw/~comb/

主辦單位：國立交通大學應用數學系

贊助單位：國科會數學研究推動中心

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2008 年組合數學 暨 新苗研討會時間表

	8月11日（星期一）		8月12日（星期二）	
			科二館 210	科二館 211
	地點: 科二館 210	Session 6	主持人:張飛黃	主持人:郭君逸
08:30-09:10	報到	08:30-08:50	連敏筠	黃志文
09:10-09:20	開幕	08:50-09:10	潘業忠	施智懷
Session 1	主持人: 傅恆霖	09:10-09:30	郭志銘	邱鈺傑
09:20-10:10	邀請演講: 陳伯亮	09:30-09:50	張嘉芬	陳子鴻
10:10-10:30	林耀仁		地點:科二館 210	
10:30-10:50	茶點	09:50-10:10	茶點	
Session 2	主持人: 陳伯亮	Session 7	主持人: 符麥克	
10:50-11:40	邀請演講: 黃國卿	10:10-11:00	邀請演講: 葉鴻國	
11:40-12:00	莊建成	11:00-11:10	休息	
12:00-13:30	午餐	Session 8	主持人: 葉鴻國	
Session 3	主持人: 黃國卿	11:10-12:00	邀請演講: 董立大	
13:30-13:50	蔡惠嬋	12:00-13:30	午餐	
13:50-14:10	郭瓊雲	Session 9	主持人: 董立大	
14:10-14:30	林陳輝	13:30-13:50	羅勝鴻	
14:30-14:50	李健源	13:50-14:10	張家榮	
14:50-15:10	茶點	14:10-14:30	林銘宏	
Session 4	主持人: 史青林	14:30-14:50	林興慶	
15:10-15:30	莊惠鈞	14:50-15:10	李建興	
15:30-15:50	洪世嘉	15:10-15:30	茶點	
15:50-16:10	林政寬	Session 10	主持人: 李國偉	
16:10-16:30	陳昆楠	15:30-15:50	張凱媛	
16:30-16:50	林和傑	15:50-16:40	邀請演講: 蕭文強	
16:50-17:00	休息	16:40-17:00	頒獎	
Session 5	主持人: 葉光清			
17:00-17:50	邀請演講: 劉德芬			
18:00--	慶祝餐會			

Contents

The Equitable Colorings of Kneser Graphs

AUTHOR: Bor-Liang Chen (陳伯亮) 09:20-10:10, August 11

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ABSTRACT

An m-coloring of a graph G is a mapping $f : V(G) \to \{1, 2, \ldots, m\}$ such that $f(x) \neq f(y)$ for any two adjacent vertices x and y in G. The chromatic number $\chi(G)$ of G is the minimum number m such that G is m-colorable. An equitable m-coloring of a graph G is an m-coloring f such that any two color classes differ in size by at most one. The equitable chromatic number $\chi_=(G)$ of G is the minimum number m such that G is equitably m-colorable. The equitable chromatic threshold $\chi_=^*(G)$ of G is the minimum number m such that G is equitably r-colorable for all $r \geq m$. It is clear that $\chi(G) \leq \chi_=(G) \leq \chi_=^*(G)$. For $n \geq 2k+1$, the Kneser graph $\mathsf{KG}(n,k)$ has the vertex set consisting of all k-subsets of an n-set. Two distinct vertices are adjacent in $\mathsf{KG}(n,k)$ if they have empty intersection as subsets. The Kneser graph $\mathsf{KG}(2k+1,k)$ is called the Odd graph, denoted by O_k. In this paper, we study the equitable colorings of Kneser graphs $\mathsf{KG}(n,k)$. Mainly, we obtain that $\chi_=(\mathsf{KG}(n,k)) \leq \chi_=^*(\mathsf{KG}(n,k)) \leq n-k+1$ and $\chi(O_k) = \chi_=(O_k) = \chi_=^*(O_k) = 3$. We also show that $\chi_=(\mathsf{KG}(n,k)) = \chi_=^*(\mathsf{KG}(n,k))$ for $k = 2$ or 3 and obtain their exact values.

KEYWORDS: equitable coloring, equitable chromatic number, equitable chromatic threshold, Kneser graph, odd graph, intersection family.

On the Anti-Ramsey Problems of Graphs

AUTHOR: Kuo-Ching Huang (黃國卿) 10:50-11:40, August 11

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ABSTRACT

Suppose that n and m are positive integers and H is a simple graph. If the edge set of K_n are colored by m colors, we can ask the following problem: *Which colorings of the subgraphs isomorphic to H in K_n must always occur*? These types problems include the *Ramsey problem*: for which n and m must a monochromatic H occur. They also include the *anti-Ramsey problem*: try to ensure a *rainbow* copy of H, that is, an H each edge of which has distinct colors. In this talk, we will talk about the anti-Ramsey problems, where H is a cycle, path, matching or star. Some other topics are also mentioned.

KEYWORDS: Ramsey, anti-Ramsey, edge-coloring, monochromatic subgraph, rainbow subgraph.

Coloring Parameters of Distance Graphs

AUTHOR: Daphne Der-Fen Liu (劉德芬) 17:00-17:50, August 11

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ABSTRACT

Distance graphs were introduced by Eggleton, Erdős and Skelton in the 80's. The study was motivated by the plane coloring problem, which is to find the least number of colors needed to paint all the points on the Euclidean plane $\Re^2$ so that any two points of unit distance apart receive distinct colors. The known bounds are 4 and 7, due to Moser and Moser and Hadwiger et al.

Distance graphs are defined by reducing the vertices considered in the plane coloring problem to all integers, where the forbidden distances might go beyond only the unit distance. For a given set D of positive integers, the *distance graph* generated by D has all integers Z as the vertex set and two vertices u and v are adjacent if $|u - v| \in D$. Denote such a graph by $G(Z, D)$. The chromatic number of distance graphs for different families of distance sets has been studied by many authors.

Besides the chromatic number, the fractional chromatic number and the circular chromatic number for distance graphs have also been studied extensively in the past decade. These coloring parameters provide more information on the structure of distance graphs and are useful to determine the chromatic number of distance graphs. Moreover, these coloring parameters of distance graphs are found closely related to some problems studied in number theory and geometry, namely, the "density of integral sequences with missing differences" and the "lonely runner conjecture."

In this talk we survey research advances in the past decade on these coloring parameters of distance graphs. We also discuss on their relations to the two number theory problems, as well as common tools used in the study of distance graphs. Moreover, several open problems and conjectures will be addressed for future research.

KEYWORDS: Distance graphs, chromatic number, fractional chromatic number, circular chromatic number.

A method to obtain lower bounds for circular chromatic number

AUTHOR: Hong-Gwa Yeh (葉鴻國) 10:10-11:00, August 12

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ABSTRACT

The circular chromatic number $\chi_c(G)$ of a graph G is a very natural generalization of the concept of chromatic number $\chi(G)$, and has been studied extensively in the past decade. In this talk we present a method for bounding the circular chromatic number from below. Let ω be an acyclic orientation of a graph G. A sequence of acyclic orientations $\omega_1, \omega_2, \omega_3, \ldots$ is obtained from ω in such a way that $\omega_1 = \omega$, and ω_i $(i \geq 2)$ is obtained from ω_{i-1} by reversing the orientations of the edges incident to the sinks of w_{i-1}. This sequence is completely determined by ω, and it can be proved that there are positive integers p and M such that $\omega_i = \omega_{i+p}$ for every integer $i \geq M$. The value p at its minimum is denoted by p_ω. To bound $\chi_c(G)$ from below, the methodology we develop in this talk is based on the acyclic orientations $\omega_M, \omega_{M+1}, \cdots, \omega_{M+p_\omega-1}$ of G. Our method demonstrates for the first time the possibility of extracting some information about $\chi_c(G)$ from the period $\omega_M, \omega_{M+1}, \cdots, \omega_{M+p_\omega-1}$ to derive lower bounds for $\chi_c(G)$. To demonstrate our methodology, throughout this talk several lower bounds for circular chromatic number are derived in a somewhat unified manner. Some of these bounds are new, and some of these bounds might follow from existing theorems.

KEYWORDS: circular chromatic number, discrete event dynamic system, token game

Study from chip-firing game to cover graph

AUTHOR: Li-Da Tong (董立大) 11:10-12:00, August 12

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ABSTRACT

A chip-firing game is played on a graph G with a nonnegative integer function c from V(G) to the set of nonnegative integers. Let $v \in V(G)$. Then $c(v)$ is the number of chips on the vertex v. A fire on v is the process that each neighbor of v gets one chip from v. In the game, we restrict that a vertex v can be fired on a function c if and only if deg$(v) \geq c(v)$. The game continues as long as fires exist. In the talk, I will introduce the relations among chip-firing game, acyclic orientation, and cover graph.

KEYWORDS: Chip-firing game, acyclic orientation, cover graph.

從離散數學到數學文化

作者**:** 蕭文強

15:50-16:40, August 12

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摘要

離散數學引人入勝的特色有三: 其一者, 具體方面它可謂伸手能及, 但抽象方面它任由想像翺翔; 其二者, 應用方面它涵蓋極廣, 事例衆多; 其三者, 它的各項課題貌似不同, 卻互相密切關連, 至其底蘊, 往往歸結爲古老的數論和幾何。

李國偉教授與我對此學科均有同好, 際此研討會慶賀他的六十歲生辰, 以及向他推動離散數學研究的貢獻致敬, 本講打算敍述過去我對離散數學這個領域內一些問題的探索, 並旁及我在數學史與數學文化的「游牧生涯」。

數學史與數學文化是李國偉著力的另一個方向, 這些年來在這個方向他給我賜教良多, 我也藉著本講向他表達謝意。

A Study of IC-coloring of Graphs

AUTHOR: Yao-Ren Lin (林耀仁) 10:10-10:30, August 11

ADVISOR: Nam-Po Chiang (江南波)

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ABSTRACT

Let $G = (V, E)$ be an undirected graph with p vertices and let $M = p(p + 1)/2$. Let f be a bijective function from V to $\{1, 2, ..., p\}$. Then f is said to be a saturating labelling of G if, given any $k(1 \leq k \leq M)$, there exists a connected subgraph H of G such that $\sum_{x \in V(H)} f(x) = k$. If G possesses a saturating labelling then G is said to be *sum-saturable*.

Let $G = (V, E)$ be an undirected graph and let f be a function from V to $\mathbf{N}$. For each subgraph H of G, we define $f_s(H) = \sum_{v \epsilon V(H)} f(v)$. Then f is said to be a *IC-coloring* of G if, given any $k(1 \leq k \leq f_s(G))$ there exists a connected subgraph H of G such that $f_s(H) = k$. And the *IC-index* of G is defined to be $M(G) = max\{f_s(G)|f \textit{ is an IC-coloring of } G\}$.

We study thoroughly the sum-saturability of all non-isomorphic trees of order less than or equal to 9, and we also prove that, for each n ≥ 2, the perfect complete n-ary tree is sum-saturable. And also we study the bounds for the IC-indices of $K_n - 2e$, $K_n - 3e$ and complete tripartite graphs.

KEYWORDS: sum-saturable, IC-index, IC-coloring.

Study on Power Domination of Graphs

AUTHOR: Chien-Cheng Chuang (莊建成) 11:40-12:00, August 11

ADVISOR: Gerard Jennhwa Chang (張鎮華)

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ABSTRACT

Electric power companies monitor the state of their electric power system by placing phase measurement units (PMUs) at selected locations in the system. They want to place as few measurement devices as possible such that these devices still monitor the whole system. This problem can be considered as a variation of the domination problem in graph theory, which we call the power domination problem.

Power domination problem is defined as follows: given a graph G, a subset S is called a power dominating set if every vertex of G can be observed by S by repeatedly applying the following rules: (i) vertices in S and their neighbors are observed; (ii) if at some stage an observed vertex has exactly one unobserved neighbor, then this neighbor is observed. The purpose of the problem is to find a minimum power dominating set S of G. The minimum cardinality of a power dominating set of G is called the power domination number $\gamma_p(G)$.

In this thesis, we first determine the power domination numbers of the Cartesian product of two cycles. We then investigate the properties of co-graphs and give an algorithm for the power domination problem on co-graphs. Finally, we present a labeling algorithm for the power domination problem on trees.

KEYWORDS: Graph theory, power domination, Cartesian product, co-graphs, trees.

Problems of Perfect Multi-Secret Sharing Schemes

AUTHOR: Hui-Chan Tsai (蔡惠嬋) 13:30-13:50, August 11

ADVISOR: Justie Su-Tzu Juan (阮夙姿)

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ABSTRACT

Secret sharing was invented by Adi Shamir and George Blakely independently in 1979. A secret sharing scheme (SSS) includes two efficient algorithms (D, R). Formally, given a group of participants $P = \{P_1, P_2, \cdots, P_n\}$. Distribution algorithm D is executed by a dealer who was given a secret, computes some shares (shared key) S_i and distributes them to each participant P_i. Reconstruction algorithm R is executed by authorized subsets of participants who combine their own shares will reconstruct the secret. A subset A of P is called a qualified subset and a secret can be reconstructed if every participant in A uses his (or her) own shares and executes the reconstruction algorithm R. $\Gamma \subseteq 2^P$ is an access structure which is the set of all qualified subsets. $\Delta \subseteq 2^P$ is a prohibited structure which is the set of all non-qualified subsets.

A multi-secret sharing scheme (MSSS) is an extension of a single secret sharing scheme in which many secrets are distributed together. In general, max-improvement ratio (MaxIR) and average-improvement ratio (AvIR) are the quantities that measure how well a MSSS performs.

We divide this thesis into three parts. In first part, we propose a perfect secret sharing scheme based on general hypergraph with prohibited structures. This scheme has no public information and includes Weng's scheme as a special case. In second part, we prove that both optimal improvement ratios of a multi-secret sharing scheme can be achieved at the same time. In third part, we propose two optimal multi-secret sharing schemes with general access structures. These two schemes are more secure and efficient than PLW scheme respectively and also achieve both optimal maximum improvement ratio and optimal average improvement ratio.

中文摘要

機密配置系統 (secret sharing scheme) 最早在1979年分別由 Shamir 及 Blakely提出。透過此系統, 參與者可依據不同權限分配到有關於此機密 (secret) 的一些片段 (shares)。有資格的參與者可依他們所拿到的片段來重建此機密, 沒有資格的參與者則無法得到任何有關於此

機密的資訊。我們稱所有有資格的參與者子集合所形成的集合為授權者集合 (access structure), 而所有沒有資格的參與者子集合所形成的集合為拒絕集集合 (prohibited structure)。

多重機密配置系統 (MSSS) 是機密配置系統的一個延伸, 表示可同時處理多個機密。一般而言, 最大改善率 (maximum improvement ratio) 及平均改善率 (average improvement ratio) 為衡量一個多重機密配置系統好壞的依據。

此篇論文可分為三大部份。第一部份, 針對一般超圖 (general hypergraph) 的拒絕集, 我們提出了一個完美機密配置系統, 此系統能處理一般化的 MSSS 問題, 並且不需要公佈任何資訊。第二部份, 對於2003年 Crescenzo 所提出的最大改善率以及平均改善率之最佳猜測值, 我們提出了一個創新的多重機密配置系統, 用以證明此兩大最佳改善率可同時被達到。第三部份, 對於2006年 Pang 等學者所提出之多重機密配置系統, 我們提出了兩個改進的多重機密配置系統 (GMS1, GMS2), 分別在時間複雜度和公佈資訊量上勝於 Pang 等學者所提出的系統。同時,GMS1更達到了弱完美 (weak-perfect) 的性質。另外, 本篇論文所提出之機密配置系統皆增設了可驗證、偵測以及多次使用之功能。

KEYWORDS: Information security, secret sharing, multi-secret, hypergraph, access structure, improvement ratio.

Bounded Tolerance Representation for Maximal Outerplanar Graphs

AUTHOR: KUO.CHIUNG-YUN (郭瓊雲) 13:50-14:10, August 11

ADVISOR: 張宜武

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ABSTRACT

We prove that a 2-connected maximal outerplanar graph G is a bounded tolerance graph if and only if there is no induced subgraph S_3 of G and G has no induced subgraph S_3 if and only if G is an interval graph.

KEYWORDS: Tolerance graphs; Maximal outerplanar graphs; Interval graphs.

A Study on Measuring Distance between Two Mixture Trees

AUTHOR: Chen-Hui Lin (林陳輝) 14:10-14:30, August 11

ADVISOR: Justie Su-Tzu Juan (阮夙姿)

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ABSTRACT

Phylogenetic tree is a tree to describe the relationship of species. For example, we can know that the tiger and the cat belong to the same family from the phylogenetic tree. It can be constructed by species information. There are many methods to building phylogenetic trees. How to know two trees are similar or not and how can one describe the amount of difference between two trees? That is why we need tree comparison metric. Tree comparison metric is to measure similarity for the phylogenetic trees, and it is an important topic in bioinformation. Mixture trees is held in 2006 by Chen and Lindsay. It has more information than traditional phylogenetic tree, for example, it shows time parameter in any point of species mutation occurs. There are many proposed metrics for the trees comparison, but no one fits to solve the tree comparison between two mixture trees up to now. We are interesting how similar between two mixture trees is. In this thesis, we define a new metric, mixture distance, to measure similarity between two mixture trees at first. Then, we develop an algorithm in time $O(n^2)$ for mixture distance and improve the algorithm to time $O(n \log n)$. Secondly, we also modify the matching distance, that is a metric for traditional phylogenetic trees, and get another new metric, mixture-matching distance, that will more fit to measure the distance between two mixture trees. Also we give an algorithm in time $O(n)$ for calculating the mixture-matching distance between two mixture trees.

中文摘要

演化樹是一種描述物種演化關係的樹狀圖, 在圖上我們可以看出老虎跟貓的祖先是同一種生物。這樣的樹可以經由物種資訊計算得到, 藉由電腦的高速計算我們可以得到大量計算出來的演化樹, 在統計學上二棵樹之間的相似程度是值得被討論的。樹狀圖的比較是測量兩樹狀圖的相似程度, 這是在生物資訊上重要的議題。混合樹是在2006年由 Chen 和 Lindsay 提出的重要演化樹。混合樹帶有比傳統的演化樹更多的資訊, 例如突變點的時間、突變的方式。我們感興趣的是混合樹之間的相似程度。雖然已經有很多測量二個樹狀圖之間距離的方法被提

出, 但是至今仍沒有一個適合用以比較兩棵混合樹。在本篇論文中, 我們首先定義了一個新的測量混合樹之間距離的方式-混合距離, 並且發展了一個 $O(n^2)$ 的演算法。接著改進這個演算法使其時間複雜度縮減為 $O(n \log n)$。

其次本論文修改了前人所定義的配對距離, 這是用以測量二個樹狀圖之間的距離方法。改進後的混合一配對距離, 將可用以測量 兩棵混合樹之間的距離。同樣地, 我們也給了一計算兩棵混合樹之混合一配對距離的演算法並且維持其時間複雜度仍是 $O(n)$。

KEYWORDS: phylogenetic tree, evolutionary tree, mixture tree, distance, tree comparison, matching distance.

Distance Three Labelings on Paths and Cycles

AUTHOR: Jian-Yuan Li (李健源) 14:30-14:50, August 11

ADVISOR: R.K. Yeh (葉光清)

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ABSTRACT

Given nonnegative integers $p \geq q \geq r$ and a graph G, an $L(p,q,r)$-labeling of G is a function $f : V(G) \rightarrow \{0,1,2,...\}$ such that (1) $|f(u) - f(v)| \geq p$ for $uv \in E(G)$, (2) $|f(u) - f(v)| \geq q$ whenever the distance between u and v in G is two and (3) $|f(u) - f(v)| \geq r$ whenever the distance between u and v is three in G. The smallest number k so that there is an $L(p,q,r)$-labeling with the maximum value k is called the $L(p,q,r)$-number of G and is denoted by $\lambda(G; p,q,r)$. This thesis studies the labeling with $r = 1$ on path and cycle.

KEYWORDS: Distance Labeling.

Mutually Independent Hamiltonian Cycles in Dual-Cube Extensive Networks

AUTHOR: Hui-Chun Chuang (莊惠鈞) 15:10-15:30, August 11

ADVISOR: Shin-Shin Kao (高欣欣)

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ABSTRACT

Dual-cubes (DC_n's), introduced by Li and Peng in 2000, are shown to be superior to hypercubes (Q_n's) in many aspects. For example, it is proved that even though DC_n and Q_{2n+1} have the number of vertices and their diameters are almost the same, DC_n consists of nearly half the number of edge of Q_{2n+1}. In 2008, Chen and Kao introduced a new kind of graphs, called *dual-cube extensive networks* (DCEN's), based on the structure of DC's. Instead of using the hypercube Q_n as a basic component for any DCEN as in dual-cubes, DCEN takes any graph G as the basic component and is then obtained by the similar construction scheme as in dual-cubes. In this paper, we will prove that the n-dimensional dual-cube contains $n + 1$ mutually independent hamiltonian cycles for $n \geq 2$. Furthermore, if any nonbipartite graph (resp. any bipartite graph) G contains n mutually independent hamiltonian cycles and is hamiltonian connected (resp. hamiltonian laceable), then $DCEN(G)$ contains at least $n + 1$ mutually independent hamiltonian cycles.

KEYWORDS: hypercube, dual-cube, hamiltonian cycle, hamiltonian connected, mutually independent.

On Channel Assignment Of Graphs

AUTHOR: Hsin-Ju Wu (洪世嘉) 15:30-15:50, August 11

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ABSTRACT

The frequency assignment problem is finding the minimum range of frequencies needed for all transmitters in the whole area. In general, a $k - L(p,q)$ labeling f for a given graph $G = (V, E)$ with positive integers p and q where $p > q$, is a function $f : V \longrightarrow \{1, 2, \cdots, k\}$ such that $|f(x) - f(y)| \geq p$ if $d(x, y) = 1$, and $|f(x) - f(y)| \geq q$ if $d(x, y) = 2$ where $d(x, y)$ is the distance between vertices x and y. The $L(p, q)$ *labeling number* $\lambda_{p,q}G$ of G is the minimum k such that there exists a $k - L(p, q)$ labeling of graph G. The $L(p, q)$ *labeling problem* is finding the $L(p, q)$ labeling number of graphs which has been proved to be NP-Complete. This thesis not only established the $L(d, 1)$ labeling number of some graphs but introduced *on-line* $L(2, 1)$-labeling module and provided some labeling algorithms to achieve on-line $L(2, 1)$-labeling number of some graphs.

KEYWORDS: $L(p, q)$ labeling, Channel assignment, Online $L(2, 1)$-labeling.

Hamilton cycles with require almost perfect matchings in hypercubes[1,2]

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15:50-16:10, August 11

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ABSTRACT

The matching P is called a perfecting matching if it contains all the vertices of this graph and is called an almost perfect matching if it consists $\lceil \frac{|V(G)|}{2} \rceil - 1$ edges. R. Škrekovski conjectured that every matching of n-dimensional hypercube with $n \geq 2$ can be extended to a Hamilton cycle. Fink [J. Fink, Perfect matchings extend to Hamilton cycles in hypercubes, J. Combin. Theory Ser. B 97 (2007) 1074-1076] proved that for any perfect matching of the n-dimensional hypercube with $n \geq 2$ it is contained by some Hamilton cycle. We prove that every almost perfect matching of n-dimensional hypercube with $n \geq 2$ can be extended to a Hamilton cycle.

KEYWORDS: Hamilton cycle; Hamilton path; Perfect matching; Almost perfect matching; Hypercube.

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完全二分圖的P_t-因子分解的探討

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ABSTRACT

假設F, G, H爲三個圖, 若H爲G的一個生成子圖, 且H中的每個分支都與F同構, 則稱G有一個F-因子. 令G和F爲兩個圖, 若G可分割成$G_1, G_2, ..., G_n$, 且每個G_i均爲G的F-因子, 則稱G有F-因子分解. 在論文中, 我們探討$K_{m,n}$的P_t-因子分解問題時, 將t分爲偶數和奇數 來探討. 首先, 當t爲偶數時, 我們分別得到 (1) 若m爲正整數, 則$K_{m,m}$有P_2-因子分解. (2) 當t爲大於或等於2的正整數時, 若$K_{m,n}$有P_t-因子分解, 則對於每一個正整數s, $K_{ms,ns}$有P_t-因子分解. (3)$K_{m,n}$有P_{2k}-因子分解的充分必要條件爲$m=n$且 $m \equiv 0 \pmod{k(2k-1)}$. 最後, 當$t$爲奇數時, 我們分別獲得 (1) 若$K$爲奇數時, 對於所有正整數$s$, 則 $K_{ks,(k+1)s}$ 有 P_{2k+1}-因子分解. (2) 若K爲奇數時, 對於所有正整數s, 則 $K_{2ks,2(k+1)s}$ 有 P_{2k+1}-因子分解. (3)$K_{m,m}$有P_{2k+1}-因子分解的充分必要條件爲 $m \equiv 0 \pmod{4k(2k+1)}$.

KEYWORDS: 完全二分圖; 路徑; 因子; 因子分解

The Linear 5-Arboricity of Complete Multipartite Graphs

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ABSTRACT

If $G_1, G_2, \ldots, G_t$ are subgraphs of a graph G such that each edge of G appears in exactly one of the subgraphs, then we say that G can be *decomposed* into $G_1, G_2, \ldots, G_t$. Besides, a *linear k-forest* is a graph whose components are paths of lengths at most k. The *linear k-arboricity* of G, denoted $la_k(G)$, is the minimum number of linear k-forests needed to decompose G. Thus the *linear k-arboricity problem* is to determine $la_k(G)$ when a graph G is given, which can be regarded as a type of graph decomposition problems.

When it comes to the linear k-arboricity problem, much attention has been focused on its two extremities. First, if k is infinite, that is, there is no length constraints on every path, then certain problem is called the *linear arboricity problem.* Second, if k is 1, then certain problem is equivalent to the so-called *edge coloring problem.* This is because a linear 1-forest is actually a matching and it can be assigned the same color. As for other values of k, there is only a small amount of related literature.

The main purpose of the research is to determine *the linear 5-arboricities of complete multipartite graphs.* To date, the literature of the linear k-arboricity problem on complete multipartite graphs is mostly concerned with cases when k is large and cases when k is 2 or 3. When given a complete multipartite graph, we shall search a lower bound and an upper bound of its linear 5-arboricity. The former can be easily obtained following the definition of linear 5-arboricity, while the latter can be achieved by using distinct techniques and approaches. Eventually, when certain lower bound equals certain upper bound, its linear 5-arboricity shall therefore be determined.

The thesis is divided into six chapters, which explain in detail the related terminology, a brief history and a literature review on the linear k-arboricity problem, and our findings. We manage to determine the linear 5-arboricities of all balanced complete bipartite graphs, partial balanced complete tripartite graphs, and other balanced complete multipartite graphs.

KEYWORDS: linear 5-arboricity, linear k-arboricity, complete multipartite graph

Cycle Cover of Graphs

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ABSTRACT

A *cycle cover* of a graph G is a collection of cycles of G which covers all edges of G. The *size* of a cycle cover is the sum of the lengths of the cycles in the cover. A *flow* in G under orientation D is an integer-valued function ϕ on $E(G)$ such that the output value $\sum_{e\in E^+(v)} \phi(e)$ is equal to the input value $\sum_{e\in E^-(v)} \phi(e)$ for each $v \in V(G)$. The *support* of ϕ is defined by $S(\phi) = \{e \in E(G) : \phi(e) \neq 0\}$. For a positive integer k, if $-k < \phi(e) < k$ for every $e \in E(G)$, then ϕ is called a *k-flow*, and furthermore, if $S(\phi) = E(G)$, then ϕ is called a *nowhere-zero k-flow*. In this thesis we prove: (1) if Tutte's 3-Flow Conjecture is true, then every $(k-1)$-edge-connected graph G with $\delta(G) = k$ has a nowhere-zero 6-flow ϕ such that when k is odd $|E_{odd}(\phi)| \geq \frac{k-1}{k}|E(G)|$ and when k is even $|E_{odd}(\phi)| \geq \frac{k-2}{k-1}|E(G)|$; (2) If a $(k-1)$-edge-connected graph G with $\delta(G) = k$ has a nowhere-zero 6-flow ϕ such that when k is odd $|E_{odd}(\phi)| \geq \frac{k-1}{k}|E(G)|$, then G has a cycle cover in which the size of the cycle cover is at most $\frac{13k+5}{9k}|E(G)|$ and when k is even $|E_{odd}(\phi)| \geq \frac{k-2}{k-1}|E(G)|$, then G has a cycle cover in which the size of the cycle cover is at most $\frac{13k-8}{9(k-1)}|E(G)|$, where $E_{odd}(\phi)=\{e \in E(G) : \phi(e) \text{ is odd }\}$.

KEYWORDS: Cycle cover, integer flow.

On the Crosstalk-free Rearrangeability of Combined Optical Multistage Interconnection Networks

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ABSTRACT

Rearrangeability of a multistage interconnection network (MIN) is that if the MIN can connect its N inputs to its N outputs in all $N!$ possible ways, by rearranging the existing connections if required. Das formulated an elegant sufficient condition for the rearrangeability of a combined $(2n-1)$-stage MIN, where $n = \log_2 N$, and presented an $O(Nn)$-time routing algorithm for MINs that satisfy the sufficient condition. However, the above definition of rearrangeability and the results of Das are for electronic MINs. Recently, optical MINs have become a promising network choice for their high performance. The fundamental difference between an electronic MIN and an optical MIN is that: two routing requests in an electronic MIN can be sent simultaneously if they are link-disjoint, while two routing requests in an optical MIN can be sent simultaneously only when their routing paths are node-disjoint, meaning that these two paths do not pass through the same switching element and therefore there is no crosstalk problem. The purpose of this thesis is to redo the works of Das for optical MINs. In particular, we formulate a sufficient condition for the crosstalk-free rearrangeability of a combined $(2n-2)$-stage and a combined $(2n-1)$-stage optical MIN, we propose an $O(Nn)$-time routing algorithm for optical MINs that satisfy the sufficient condition. In this thesis we also propose an algorithm to realize any permutation in a baseline network with node-disjoint paths in four passes.

KEYWORDS: Multistage interconnection network; Optical multistage interconnection network; Rearrangeability; Permutation routing; Crosstalk; Benes network; Baseline network; Reverse baseline network

Triangle-free Distance-regular Graphs with Pentagons

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ABSTRACT

Let Γ denote a distance-regular graph with Q-polynomial property. Assume the diameter D of Γ is at least 3 and the intersection numbers $a_1 = 0$ and $a_2 \neq 0$. We show the following (i)-(iii) are equivalent.

(i) Γ is Q-polynomial and contains no parallelograms of length 3.

(ii) Γ is Q-polynomial and contains no parallelograms of any length i for $3 \leq i \leq D$.

(iii) Γ has classical parameters (D, b, α, β) for some real constants b, α, β with $b < -1$.

When (i)-(iii) hold, we show that Γ has 3-bounded property. Using this property we prove that the intersection number c_2 is either 1 or 2, and if $c_2 = 1$ then $(b, \alpha, \beta) = (-2, -2, ((-2)^{D+1} - 1)/3)$.

KEYWORDS: Distance-regular graph, Q-polynomial, classical parameters.

Learning a Hidden Graph with Adaptive Algorithm

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ABSTRACT

We consider the problem of learning a hidden graph using edge-detecting queries in a model where the only allowed operation is to query whether a set of vertices induces an edge of the hidden graph or not. In [V. Grebinski and G. Kucherov, Optimal query bounds for reconstructing a Hamiltonian cycle in complete graphs, In fifth Israel symposium on the Theory of Computing Systems, 166–173, 1997.], Grebinski and Kucherov give a deterministic adaptive algorithm for learning Hamiltonian cycles using $O(n \log n)$ queries. In [R. Beigel, N. Alon, S. Kasif, M. S. Apaydin and L. Fortnow, An optimal procedure for gap closing in whole genome shotgun sequencing, In RECOMB, 22–30, 2001.], Beigel et al. describe an 8-round deterministic algorithm for learning matchings using $O(n \log n)$ queries, which has direct application in genome sequencing projects. In [D. Angluin and J. Chen. Learning a hidden graph using $O(\log n)$ queries per edge. Manuscript, 2006.], Angluin and Chen use at most $12m \log n$ queries in their algorithm for learning a general graph. In this thesis we present an adaptive algorithm that learns a general graph with n vertices and m edges using at most $(2 \log n + 9)m$ queries.

KEYWORDS: hidden graph; genome shotgun sequencing; edge-detecting queries

Diameters and Wide-Diameters of de Bruijn Graphs

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ABSTRACT

In graph theory, the study of fault tolerance and transmission delay of networks, the connectivity and diameter of a graph are two very important parameters. Since the de Bruijn graphs and generalized de Bruijn graphs are known to have small diameters, and simple routing strategies, they have been widely used as models for communication networks and multiprocessor systems.

The directed de Bruijn graph $B(d,n)$ has vertex-set $V = \{x_1x_2\cdots x_n : x_i \in Z_d, i = 1,2,\cdots,n\}$ and directed edge-set E, where for $\mathbf{x} = x_1x_2\ldots x_n$, $\mathbf{y} = y_1y_2\ldots y_n \in V$, $\mathbf{xy} \in E$ if and only if $y_i = x_{i+1}$ for $i = 1,2,\ldots,n-1$. Clearly, $B(d,n)$ has d^n vertices thus there is a restriction on the number of vertices. To conquer this disadvantage, a modification, generalized de Bruijn graphs, was obtained later by Imase and Itoh, and independently by Reddy, Pradhadn and Kuhl.

The generalized directed de Bruijn graph $G_B(n,m)$ is a directed graph whose vertices are $0,1,2,\cdots,m-1$ and the directed edges are of the form

$$i \to in+\alpha \pmod m, \forall i \in \{0,1,\ldots,m-1\} \text{ and } \forall \alpha \in \{0,1,\ldots,n-1\}.$$

Then, by replacing directed edges with undirected edges and omitting the loops and multi-edges of the directed de Bruijn graphs and generalized directed de Bruijn graphs, we have the undirected de Bruijn graphs and generalized undirected de Bruijn graphs respectively.

In this thesis, we study the wide-diameters of undirected de Bruijn graphs, and study the diameters of generalized undirected de Bruijn graphs.

KEYWORDS: wide-diameter, diameter, de Bruijn graphs, generalized de Bruijn graphs.

Optimal All-to-All Personalized Exchange Algorithms in Generalized Shuffle-Exchange Networks

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ABSTRACT

Previous all-to-all personalized exchange algorithms are mainly for hypercube, mesh, and torus. In [Y. Yang, J. Wang, "Optimal all-to-all personalized exchange in self-routable multistage networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 11, no. 3, pp. 261-274, 2000.], Yang and Wang first proposed an all-to-all personalized exchange algorithm for multistage interconnection networks (MINs). Their algorithm is optimal and works for a class of unique-path, self-routable MINs (for example, baseline, omega, banyan networks). Do notice that all the MINs considered in Yang and Wang's paper must have the unique-path property and must satisfy $N = 2^{n+1}$, in which N is the number of inputs (outputs), 2 means all the switches are of size 2×2, and $n + 1$ is the number of stages in the MINs. To our knowledge, no one has studied all-to-all personalized exchange in MINs which do not have the unique-path property and do not satisfy $N = 2^{n+1}$. In [K. Padmanabham, "Design and analysis of even-sized binary shuffle-exchange networks for multiprocessors," *IEEE Trans. Parallel Distrib. Syst.*, vol. 2, no. 4, pp. 385-397, Oct. 1991.], Padmanabhan proposed the generalized shuffle-exchange network (GSEN), which allows $N \neq 2^{n+1}$ (thus N can be any even number). A GSEN becomes an omega network (i.e., the shuffle-exchange network) when $N = 2^{n+1}$. Since a GSEN is not necessarily a unique-path MIN, Yang and Wang's optimal algorithm may not apply. The purpose of this thesis is to propose two optimal all-to-all personalized exchange algorithms for GSENs. Unlike Yang and Wang's algorithm, we abandon the the requirement on the unique-path. The first algorithm uses the stage control technique and works for all even N. We will prove it is optimal when the stage control technique is assumed. On the contrary, the second algorithm does not use the stage control technique and works for all N such that $N \equiv 2 \pmod 4$. We will prove that it is optimal.

KEYWORDS: Multistage interconnection network; Shuffle-exchange network; Omega network; Parallel and distributed computing; All-to-all communication; All-to-all personalized exchange.

Total Relative Displacements in Graphs

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ABSTRACT

Let f be a permutation of $V(G)$. Define $\delta_f(x,y) = |d_G(x,y) - d_G(f(x), f(y))|$ and the *total relative displacements of permutation f in G*, $\delta_f(G) = \sum \delta_f(x,y)$, over all the unordered pairs $\{x,y\}$ of distinct vertices of G. The smallest positive value of $\delta_f(G)$ among all the permutations f of $V(G)$ is denoted by $\pi(G)$ and the maximum value of $\delta_f(G)$ among all the permutations f of $V(G)$ is denoted by $\pi^*(G)$. The permutation f with $\delta_f(G) = \pi(G)$ is called a near automorphism of G and the permutation g with $\delta_g(G) = \pi^*(G)$ is called a chaotic mapping of G.

This thesis is devoted to investigate the permutations which are near automorphisms and chaotic mappings respectively. In Chapter 1, we start with an introduction of the total relative displacement and present a short survey of the existing literature. Then, in Chapter 2, we study the graphs with small near automorphism values, and we mainly characterize certain graphs G with $\pi(G) = 2$ and trees T with $\pi(T) = 4$. In Chapter 3, our focus will be on the near automorphisms of the cycles C_n and we prove that $\pi(C_n) = 4\lfloor \frac{n}{2} \rfloor - 4$ for $n \geq 4$. We then study the trees T of order n with $\pi(T) = 2n-4$, $n \geq 3$, in Chapter 4. ($2n-4$ is the maximum total relative displacement of a near automorphism of an order n graph.) Before the end of this thesis, we also study the lower bound of $\pi^*(G)$ for some graph G. We obtain a better lower bound of paths P_n than the currently known one. Finally, we conclude this thesis with several remarks which include the direction of further study and open problems.

KEYWORDS: Automorphism, near automorphism.

Routing Permutations in the Baseline Network and in the Omega Network

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ABSTRACT

Routing permutations in a multistage interconnection network (MIN) is an important operation in parallel and distributed computing systems. Let N denote the number of inputs and outputs of a given MIN. It is well-known that an MIN may not be able to realize all the $N!$ possible permutations. A permutation is admissible in an MIN if it can be realized in that MIN. Some researchers considered adding extra hardware so that the resultant MIN can realize all the $N!$ possible permutations; see [C.-T. Lea and D.-J. Shyy, "Tradeoff of horizontal decomposition versus vertical stacking in rearrangeable nonblocking networks," *IEEE Trans. Commun.*, vol. 39, no. 6 (1991) pp. 899-904]. Other researchers considered using extra passes to realize all the $N!$ possible permutations; see [Y. Yang and J. Wang, "Routing permutations with link-disjoint and node-disjoint paths in a class of self-routable interconnects," *IEEE Trans. Parallel Distrib. Syst.*, vol. 14, no. 4 (2003) pp. 383-393; Y. Yang and J. Wang, "Routing permutations on baseline networks with node-disjoint paths," *IEEE Trans. Parallel Distrib. Syst.*, vol. 16, no. 8 (2005) pp. 737-746]. The purpose of this thesis is twofold: we propose an algorithm to determine whether a permutation is admissible in the Baseline network and an algorithm to determine whether a permutation is admissible in the Omega network; we also implement the algorithm in [Y. Yang and J. Wang, "Routing permutations on baseline networks with node-disjoint paths," *IEEE Trans. Parallel Distrib. Syst.*, vol. 16, no. 8 (2005) pp. 737-746] into a computer program.

KEYWORDS: Multistage interconnection network; Routing; Permutation; Semi-permutation; Baseline network; Omega network.

Global defensive alliances in double-loop networks

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ABSTRACT

A *defensive alliance* in graph $G = (V, E)$ is a set of vertices $S \subseteq V$ satisfying $|N[v] \cap S| \geq |N(v) \cap (V - S)|$ for any $v \in S$, $N(v) = \{u : uv \in E\}$, and $N[v] = N(v) \cup \{v\}$. Because of such an alliance, the vertices in S, agreeing to mutually support each other, have the strength of numbers to be able to defend themselves from the vertices in $V - S$. A defensive alliance S is called *global* if $N[S] = V$.

A *double-loop network* $\overrightarrow{DL}(n; a, b)$ can be viewed as a directed graph with n vertices $0, 1, 2, \ldots, (n-1)$ and $2n$ directed edges of the form $i \to i + a \pmod n$ and $i \to i + b \pmod n$, referred to as a-links and b-links. In this thesis, any reference to $DL(n; a, b)$ will mean an underlying graph of a directed graph $\overrightarrow{DL}(n; a, b)$.

In this thesis, we study global defensive alliance in $DL(n; a, b)$. We determine the value of the global defensive alliance number in $DL(n; 1, 2)$, $DL(n; 1, 3)$, $DL(3n; 1, 3k)$, and $DL(n; 1, \lfloor \frac{n}{2} \rfloor)$. Finally, we research into the relation between $\gamma_a(G)$ and integer programming for G being a k-regular graph.

KEYWORDS: alliance,double-loop networks

Triangle-free subcubic graphs with small bipartite density

AUTHOR: Chia-Jung Chang (張家榮) 13:50-14:10, August 12

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ABSTRACT

Suppose G is a graph with n vertices and m edges. Let n' be the maximum number of vertices in an induced bipartite subgraph of G and let m' be the maximum number of edges in a spanning bipartite subgraph of G. Then $b(G) = m'/m$ is called the bipartite density of G, and $b^*(G) = n'/n$ is called the bipartite ratio of G. It is proved by Zhu that if G is a 2-connected triangle-free subcubic graph, then apart from seven exceptional graphs, we have $b(G) \geq 17/21$. If G is a 2-connected triangle-free subcubic graph, then $b^*(G) \geq 5/7$ provided that G is not the Petersen graph and not the dodecahedron. These two results are consequences of a more technical result which is proved by induction: If G is a 2-connected triangle-free subcubic graph with minimum degree 2, then G has an induced bipartite subgraph H with $|V(H)| \geq (5n_3 + 6n_2 + \epsilon(G))/7$, where $n_i = n_i(G)$ are the number of degree i vertices of G, and $\epsilon(G) \in \{-2, -1, 0, 1\}$. To determine $\epsilon(G)$, four classes of graphs $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ and F-cycles are constructed. For $G \in \mathcal{G}_i$, we have $\epsilon(G) = -3 + i$ and for an F-cycle G, we have $\epsilon(G) = 0$. Otherwise, $\epsilon(G) = 1$. To construct these graph classes, eleven graph operations are used. This thesis studies the structural property of graphs in $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$. First of all, a computer algorithm is used to generate all the graphs in $\mathcal{G}_i$ for $i = 1, 2, 3$. Let $\mathcal{P}$ be the set of 2-edge connected subcubic triangle-free planar graphs with minimum degree 2. Let $\mathcal{G'}_1$ be the set of graphs in $\mathcal{P}$ with all faces of degree 5, $\mathcal{G}'_2$ the set of graphs in $\mathcal{P}$ with all faces of degree 5 except that one face has degree 7, and $\mathcal{G}'_3$ the set of graphs in $\mathcal{P}$ with all faces of degree 5 except that either two faces are of degree 7 or one face is of degree 9. By checking the graphs generated by the computer algorithm, it is easy to see that $\mathcal{G}_i \subseteq \mathcal{G}'_i$ for $i = 1, 2, 3$. The main results of this thesis are that for $i = 1, 2$, $\mathcal{G}_i = \mathcal{G}'_i$ and $\mathcal{G}'_3 = \mathcal{G}_3 \cup \mathcal{R}$, where $\mathcal{R}$ is a set of nine F-cycles.

KEYWORDS: triangle-free, subcubic, bipartite density, bipartite ratio, planar graph.

On the domination numbers of prisms of cycles

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ABSTRACT

Let $\gamma(G)$ be the domination number of a graph G. For any permutation π of the vertex set of a graph G, the prism of G with respect to π is the graph πG obtained from two copies G_1 and G_2 of G by joining $u \in V(G_1)$ and $v \in V(G_2)$ iff $v = \pi(u)$. We prove that

$$\gamma(\pi C_n) \geq \begin{cases} \frac{n}{2}, & \text{if } n = 4k,; \\ \left\lceil \frac{n+1}{2} \right\rceil, & \text{if } n \neq 4k. \end{cases} \text{ and } \gamma(\pi C_n) \leq \left\lceil \frac{2n-1}{3} \right\rceil \text{ for all } \pi.$$

We also find a permutation π_t such that $\gamma(\pi_t C_n) = k$, where k between the lower bound and the upper bound of $\gamma(\pi C_n)$ in above. Finally, we prove that if $\pi_b C_n$ is a bipartite graph, then

$$\gamma(\pi_b C_n) \geq \begin{cases} \frac{n}{2}, & \text{n = 4k;} \\ \left\lceil \frac{n+1}{2} \right\rceil, & \text{if } n = 4k+2, \end{cases} \text{ and } \gamma(\pi_b C_n) \leq \left\lfloor \frac{5n+2}{8} \right\rfloor.$$

KEYWORDS: domination number, prism, cycle.

Vertex ranking numbers of graphs

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Given a graph G, a *vertex ranking* of G is a mapping f from $V(G)$ to the set of all natural numbers, such that for any path between two distinct vertices u and v with $f(u) = f(v)$, there is a vertex w in the path with $f(w) > f(u)$. If f is a vertex ranking of G, the *ranking number of* G *under* f, denoted $\gamma_f(G)$, is defined by $\gamma_f(G) = \max\{f(v) : v \in V(G)\}$, and the *vertex ranking number* of G, denoted $\gamma(G)$, is defined by $\gamma(G) = \min\{\gamma_f(G) : f \text{ is a vertex ranking of } G\}$. The *vertex ranking problem* is to determine the vertex ranking number $r(G)$ of a given graph G. This problem is a nature model for the manufacturing scheduling problem. We study the vertex ranking numbers of graphs in this thesis. We consider the relation between the vertex ranking numbers and the minimal cut sets, and the relation between the vertex ranking numbers and the independent sets of graphs. From this, we obtain the vertex ranking numbers of the powers of paths and the powers of cycles, the Cartesian product of P_2 with P_n or C_n, and the caterpilars. And we also find the vertex ranking numbers of the composition of two graphs and corona of two graphs in this thesis.

Bounded-flow transmission problem of graphs

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Given a graph G and a set $S \subseteq V(G)$ together with a set $M_s = \{m(v) : v \in S\}$, the a *bounded-k broadcasting number of G corresponding to* (S, M_S), denoted $b_k(G; S; M_S)$, is the minimum number of time needed to complete the broadcasting from S, that is, to let all the vertices in G know all the messages in $\bigcup_{v \in S} m(v)$, subject to the constraints that at each time unit, the number of messages that can be interchanged between any two vertices u and v in G, with $uv \in E(G)$, is bounded by a constant k. We want to find the minimum number of time units required to complete the transmittion, that is, to let all the vertices in G know all the messages. We call such a problem a *bounded-k broadcasting problem*. Given a graph G, the *bounded-k broadcasting number of* G, denoted $b_k(G)$, is the number $b_k(G; S; M_S)$, under the condition that $S = V(G)$, $|m(v)| = 1$, $m(u) \cap m(v) = \emptyset$ for all $u, v \in V(G)$, $u \neq v$, and $M_S = \{m(v) : v \in V(G)\}$. Clearly, the bounded-1 broadcasting problem is just the multi-message, multi-originator problem, and the bounded-k broadcasting problem is the same as the gossiping problem for any graph G with $|V(G)| \leq k$. Hence this problem can be viewed as a generalization of both the broadcasting problem and the gossiping problem. In this thesis, we give some lower bounds for the bounded-k-broadcasting number of G, and find the bounded-2-broadcasting numbers of C_n and the bounded-2-broadcasting numbers of P_m for all $n \geq 3$ and $m \geq 2$. And from this, we obtain the bounded-2-broadcasting number of G when G is Hamiltonian, and deduce an upper bound for the bounded-2-broadcasting number of G when G has a Hamiltonian path. We also consider the bounded-2-broadcasting numbers of complete bipartite graphs $K_{m,n}$ with $m, n \geq 2$, which is not Hamiltonian when $m \neq n$.

Distance-two domination of double-loop networks

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ABSTRACT

Due to a practically resource sharing problem, we consider a variation of the domination problem in this thesis which we call the *distance-two domination problem*. We collect a vertex subset D that every vertex v in D can get a complete resource, using 3 to weight. Moreover, the vertex v in D can offer partial resource, using 2 to weight, to each neighbor of v, offer little resource, using 1 to weight, to each distance-two neighbor of v, and offer no resource, using 0 to weight, to others vertices which have the distance more than 2 with v. For each vertex v in the graph, we define the weighted function with respect to the vertex subset D by $w_D(v) = 3|v \cap D| + 2|N(v) \cap D| + |N_2(v) \cap D|$. For a graph $G = (V, E)$, we say that $D \subseteq V$ is a $D_{3,2,1}$-dominating set of G if and only if $w_D(v) \geq 3$ for all $v \in V$. The $D_{3,2,1}$-*domination number* $\gamma_{3,2,1}(G)$ of a graph G is the minimum cardinality of a $D_{3,2,1}$-dominating set of G.

A *double-loop network* $\overrightarrow{DL}(n; a, b)$ can be represented as a directed graph with n vertices $\{0, 1, ..., n-1\}$ and $2n$ directed edges of the form $i \to i + a \pmod n$, and $i \to i + b \pmod n$, referred to as a-links and b-links. In this thesis, we discuss the double-loop network without the direction which is denoted by $DL(n; a, b)$.

This thesis is organized as follows. Section 1 gives basic definitions and notation. Section 2 investigates the distance-two domination of $DL(n, 1, 2)$. Section 3 investigates the distance-two domination of $DL(n, 1, 3)$. Section 4 investigates the distance-two domination of $DL(n, 1, \lfloor \frac{n}{2} \rfloor)$. We provide the integer programming method to canvass $\gamma_{3,2,1}(G)$ in the final section.

KEYWORDS: dominating set, domination number, weighted function, $D_{3,2,1}$-domination number, $D_{3,2,1}$-dominating set, double-loop network

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